HEIGHTS AND DISTANCES

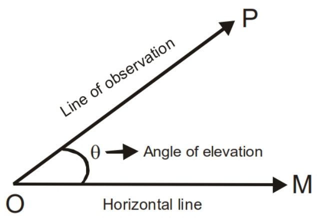
INTRODUCTION

One of the important application of trigonometry is in finding the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

DEFINITIONS

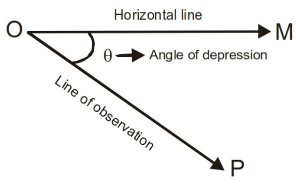
Angle of elevation

Let O and P be two points where P is at a Higher level than O. Let O be at the position of the observer and P be the position of the object. Draw a horizontal line OM through the point O. OP is called the line of observation or line of sight. Then $\angle POM = \theta$ is called the angle of elevation of P as observed from O.



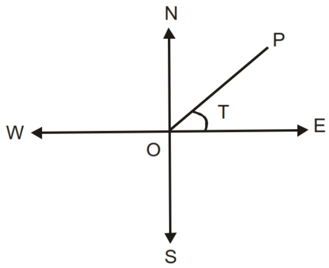
Angle of depression

In the above figure, if P be at a lower level than O, then $\angle MOP = \theta$ is called the angle of depression.



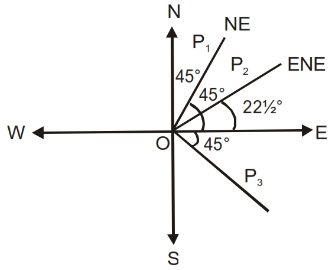
Bearing

In the figure, if the observer and the object i.e., O and P be on the same level then bearing is defined. To measure the 'Bearing', the four standard directions East, West, North and South are taken as the cardinal directions. Angle between the line of observation i.e. OP and any one standard direction - east, west, north or south is measured. Thus, $\angle POE = \theta$ is called the bearing of point P with respect to O measured from east to north. In other words the bearing of P as seen from O is the direction in which P is seen from O .



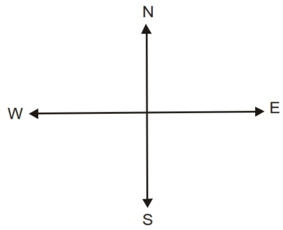
North - east

North - east means equally inclined to north and east, south -east means equally inclined to south and east. ENE means equally inclined to east and north east



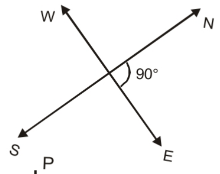
Problems on two dimension

If the total actual figure is located in one plane, the problem is of two dimensions. For direction in two dimensional figures, cross vertically as shown in the figure.

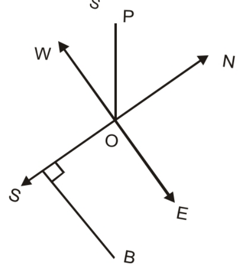


Problems on three dimension

If total actual figure is located in more than one plane, the problem will be of three dimensions for direction in three dimensional figures, cross obliquely as shown. Clearly this oblique cross represents the horizontal plane.



If OP be a vertical tower perpendicular to the plane, then it will be represented like the figure clearly $\angle POA = 90°$ if the observer at A moves in east direction. We draw a line AB parallel to east to represent this movement. Clearly $\angle OAB = 90°$ (angle between north and east)

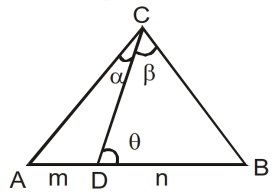


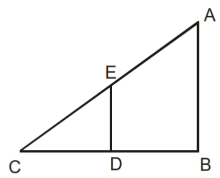
SOME USEFUL RESULT

m–n theorem : In a triangle, if m,n, θ, α, β are as shown in the diagram, then

$(m + n) \cot \theta = m \cot \alpha - n \cot \beta$

$(m + n) \cot \theta = n \cot A - m \cot B$

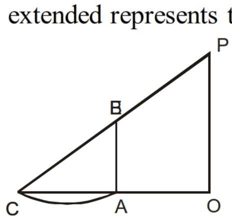




In a triangle ABC, if DE || AB.

then , $\frac{AB}{DE} = \frac{BC}{DC}$

To find the shadow of line object AB with respect to the light source P, we first join the upper points P and B. Let O be the projection of P on the plane on which object AB is situated join OA. The section AC obtained by the intersection of the lines PB and OA. extended represents the shadow of AB with respect to light source P.

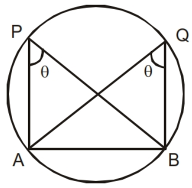


PROBLEMS BASED ON PROPERTIES OF A CIRCLE

Whenever a line subtends equal angles at two points or the greatest angles at some points on a given line, such problems can be solved easily using the properties of a circle. Mainly the following geometrical properties of a circle will be used

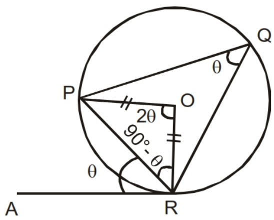
Angles on the same segment of a circle are equal.

In other words, we can say that if the angles APB and AQB subtended on the segment AB are equal, a circle will pass through the points A,B,Q and P i.e., the point A,B,Q and P will be concyclic.



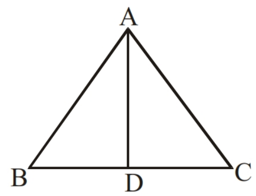
If AR be the tangent to the circle through the points P,Q,R then $\angle PRA = \angle PQR = \theta$.

[angle between any chord and the tangent to the circle is equal to the angle subendend by the chord at the circumference]



Apollonius theorem : In a triangle ABC, AD is median, then

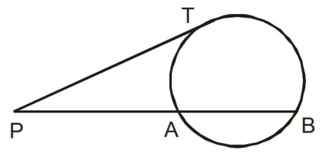
$AB^2 + AC^2 = 2\ (AD^2 + BD^2)$



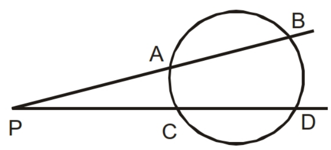
If AD is angle bisector of triangle ABC at A, then

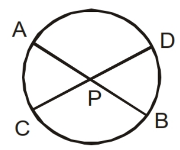
$$\frac{AB}{AC} = \frac{BD}{DC} = \frac{c}{b}$$

$PA.PB = PT^2$



$PA.PB = PC.PD$





If AB subtends angle a at C, then AB makes same angle a with tangent at point B. i.e., ĐABD = a